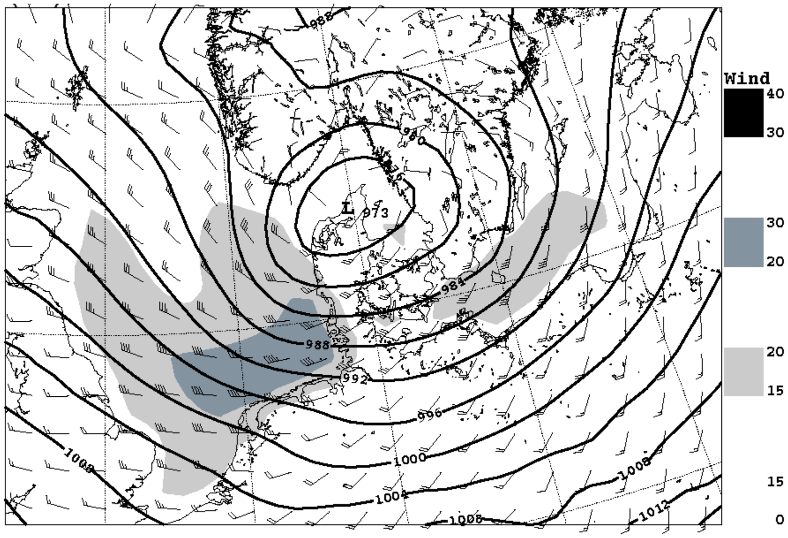
A mass-conservative version of the semi-Lagrangian semi-implicit HIRLAM using Lagrangian vertical coordinates



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National Center for Atmospheric Research

4th Workshop on the Use of Isentropic & other Quasi-Lagrangian Vertical Coordinates in Atmosphere & Ocean Modeling
7-9 October, 2008, NOAA, Boulder, Colorado

NCAR ESSL's Climate & Global Dynamics

Outline

- What is HIRLAM? (non-conservative semi-implicit semi-Lagrangian weather forecast model) …

- How to make semi-implicit semi-Lagrangian models mass-conservative:

  - Horizontal advection: Finite-volume Semi-Lagrangian schemes (a.k.a. Cell-Integral Semi-Lagrangian CISL)

  - Semi-implicit time-stepping with CISL schemes

  - Vertical problem:

    * Floating Lagrangian coordinates during each time-step; consistent splitting of the horizontal and vertical problems
    * Lagrangian treatment of energy conversion term

- Preliminary tests (adiabatic and full physics)

About Hirlam Programme

The international programme HIRLAM , HIgh Resolution Limited Area Model, is a cooperation of the following meteorological institutes:

- Danish Meteorological Institute (DMI) (Denmark)
- Estonian Meteorological and Hydrological Institute (EMHI) (Estonia)
- Finnish Meteorological Institute (FMI) (Finland)
- Icelandic Meteorological Office (VI) (Iceland)
- Irish Meteorological Service (IMS) (Ireland)
- Royal Netherlands Meteorological Institute (KNMI) (The Netherlands)
- The Norwegian Meteorological Institute (met.no) (Norway)
- Spanish Meteorological Institute (INM) (Spain)
- Swedish Meteorological and Hydrological Institute (SMHI) (Sweden)

In addition, there is a research cooperation with Météo-France (France).

The aim of the Hirlam programme is to develop and maintain a numerical short-range weather forecasting system for operational use by the participating institutes. The programme was initiated in 1985 and has gone through numerous phasesin the past two decades. Since 1 January 2006, the programme enters its new phase, HIRLAM-A. The HIRLAM forecast system is now used in routine weather forecasting at DMI, FMI, IMS, KNMI, met.no, INM, and SMHI. The HIRLAM programme is controlled by the HIRLAM council, which consists of the directors of the participating institutes. The programme is managed by the management group consisting of:

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Traditional semi-Lagrangian semi-implicit grid-point model.

Non-conservative formulation

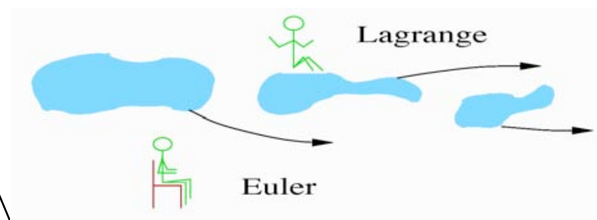
Part I:

Horizontal problem

Cell-integrated semi-Lagrangian schemes

Finite Volume schemes

- (semi-)Lagrangian
  - fully 2D
  - 1D sweeps (cascade)
- Eulerian
  - fully 2D
  - dimensional split



Eulerian finite-volume scheme

Integrate the flux-form Eulerian continuity equation

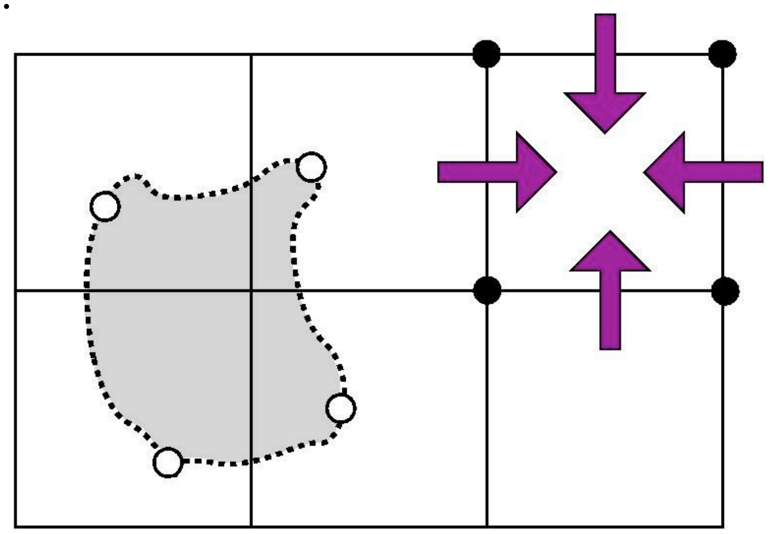
$$\boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot (\vec{v}\rho) = 0}$$

over the arrival area ΔA an apply Gauss's divergence theorem:

$$\frac{\partial}{\partial t}\left[\iint_{\Delta A} \rho \, dx\, dy\right] = -\iint_{\Delta A} \nabla \cdot (\rho \vec{v}) \; dx\, dy = -\iint_{\partial(\Delta A)} \rho\, \vec{v} \cdot \vec{n}\, d\ell$$

where $\vec{n}$ is the outward pointing unit normal vector of the boundary $\partial(\Delta A)$. Discretizing the left-hand side and time-averaging the right-hand side, yields:

$$\overline{\rho}^{n+1} \Delta A = \overline{\rho}^{n} \Delta A - \Delta t \sum_{i=1}^{4} \left[\overline{\overline{\langle \rho \vec{v} \rangle}} \cdot \vec{n}\, \Delta \ell\right]_i$$



where the angle brackets represent averages in the *x* or *y*-direction and the double-bar refers to the time average over the time-step Δt . So the right-hand side represents the mass transported through each of the four arrival cell faces into the cell during one time step.

Cell-integrated semi-Lagrangian (CISL) scheme

Integrate Lagrangian continuity equation over a cell/volume *A* moving with the flow:

$$\boxed{\frac{D}{Dt}\left[\iint_A \rho \, dx\, dy\right] = 0}$$

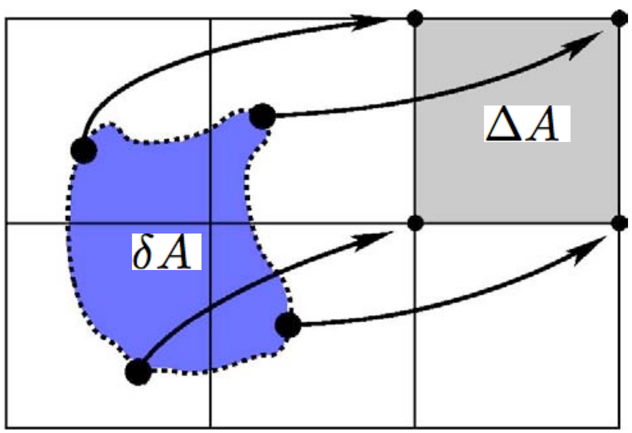
Discretizing this equation using backward trajectories, the CISL continuity equation results:

$$\boxed{\overline{\rho}^{n+1}\Delta A = \overline{\rho}_*^n \delta A}$$

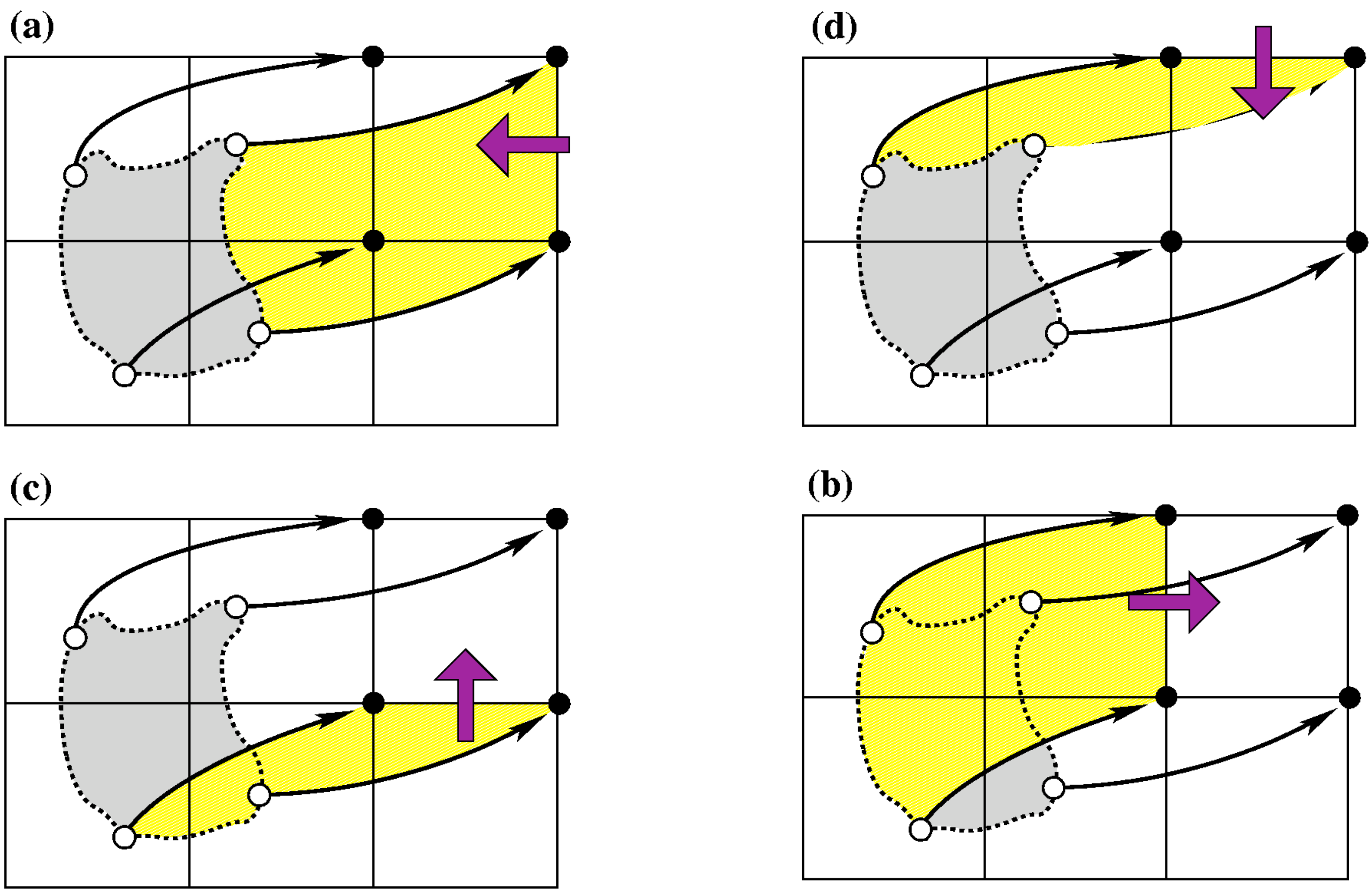
where ΔA and δA is referred to as the *departure* and *arrival* area, respectively.

$$\overline{\rho}_*^n = \frac{1}{\delta A}\iint_{\delta A} \rho^n(x,y)\, dx\, dy$$

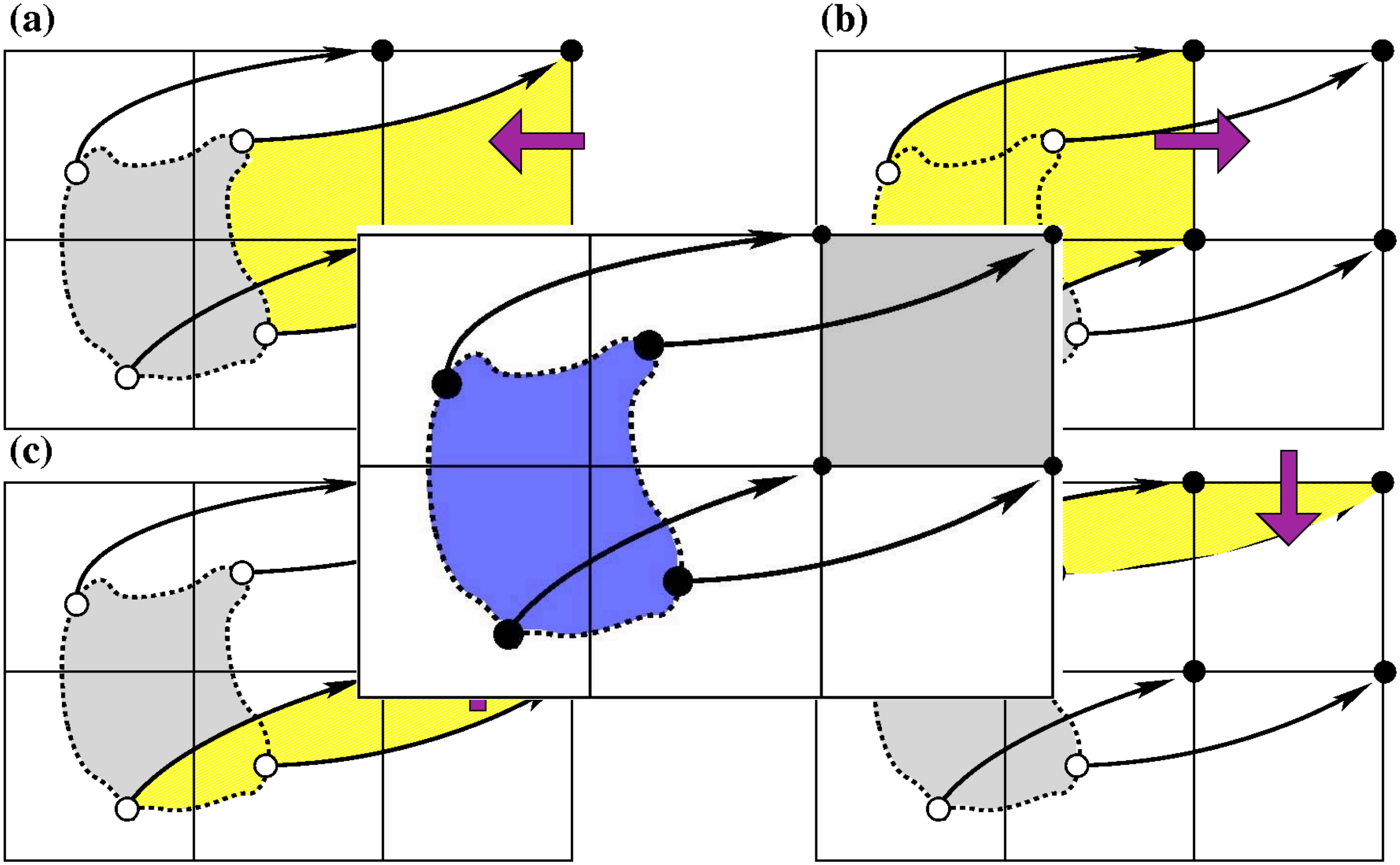
is the is the integral of $\rho^n(x,y)$ over the departure area, where $\rho^n(x,y)$ is the sub-grid-scale reconstruction.



$$\overline{\rho}^{n+1}\Delta A = \overline{\rho}^{n}\Delta A - \Delta t \sum_{i=1}^{4} m_i$$



Equivalence between Eulerian and Lagrangian finite-volume schemes



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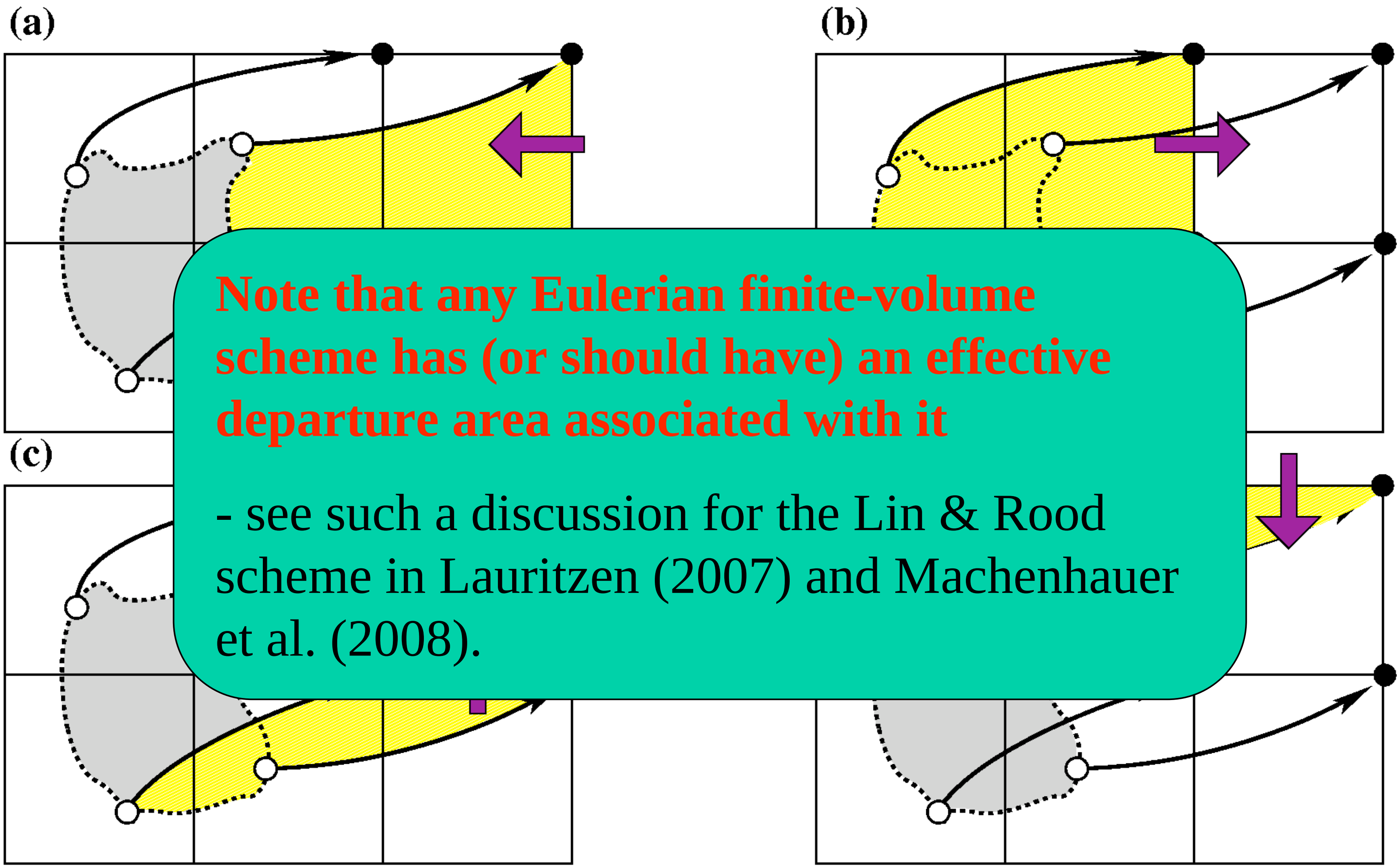


Note that any Eulerian finite-volume scheme has (or should have) an effective departure area associated with it

- see such a discussion for the Lin & Rood scheme in Lauritzen (2007) and Machenhauer et al. (2008).

A note on CISL schemes

- Accuracy of trajectories

  CISL schemes are more sensitive to accurate trajectories than grid-point semi-Lagrangian schemes since the divergence is absorbed in the trajectories (Thuburn 2008, Lauritzen et al. 2005, Kaas 2008):

  Shallow water and 3D hydrostatic tests show that the acceleration should be included in the trajectory estimation when using CISL schemes.

- - : “Geometric approach” makes it difficult to exactly preserve a constant for non-divergent flow.

- +: inherently local, allow for long time steps, reuse geometric information for each additional tracer and have monotonic options.

Part II:

Semi-implicit time-stepping with CISL schemes

Traditional semi-implicit discretization of continuity equation

$$\rho^{n+1} = \rho^{n+1}_{exp} + \frac{\Delta t}{2}\rho^{ref}\left(\tilde{D}^{n+1} - D^{n+1}\right),$$

where ρ^{ref} constant, D 'Eulerian' divergence

$$D = \nabla \cdot \vec{v},$$

and $\tilde{D}$ is the divergence calculated using only extrapolated winds.

Finite difference discretization of D leads to a linear function of $\vec{v}$
$\Rightarrow$ closing system using momentum equations leads to simple elliptic equation

Semi-implicit CISL continuity equation

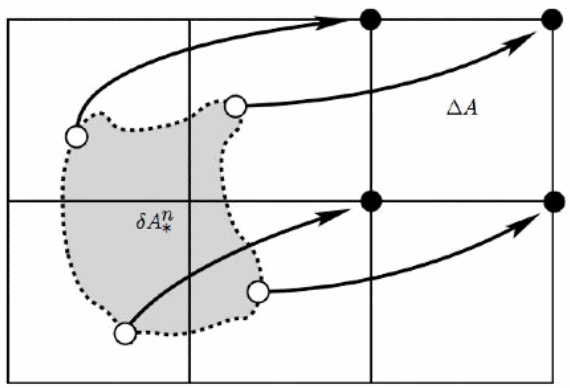
Ideally the semi-implicit cell-integrated continuity equation is given by

$$\overline{\rho}^{n+1} = \overline{\rho}^{n+1}_{exp} + \frac{\Delta t}{2}\rho^{ref}\left(\tilde{\mathbb{D}}^{n+1} - \mathbb{D}^{n+1}\right),$$

where $\mathbb{D}$ Lagrangian divergence

$$\mathbb{D} = \frac{1}{\Delta A}\frac{\Delta A - \delta A^n_*}{\Delta t}.$$

$\tilde{\mathbb{D}}$ is the divergence calculated using only extrapolated winds.



For consistency Lagrangian (not Eulerian) definition of $\mathbb{D}$ must be used!

Semi-implicit CISL continuity equation

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where $\mathbb{D}$ Lagrangian divergence

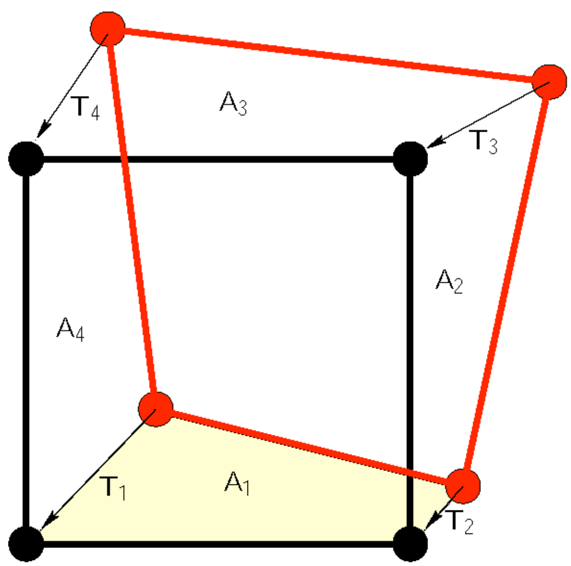
$$\mathbb{D} = \frac{1}{\Delta A}\frac{\Delta A - \delta A_*^n}{\Delta t}.$$

$\tilde{\mathbb{D}}$ is the divergence calculated using only extrapolated winds.

Again, to close the system $\mathbb{D} = \mathbb{D}(u, v)$. However, $\mathbb{D}$ incorporates the departure cell geometry and thus $\mathbb{D}(u, v)$ is a non-linear function of (u, v)
$\Rightarrow$ very complicated elliptic equation.

Semi-implicit CISL continuity equation

$$\mathbb{D}_{i,j}^{n+1} = \frac{1}{2}\left(\delta_x u_{ij}^{n+1} + \delta_y v_{ij}^{n+1} + \delta_x u_{ij+1}^{n+1} + \delta_y v_{i+1j}^{n+1}\right)$$
$$+ \frac{1}{2}\frac{\Delta t}{\Delta x\,\Delta y}\left[\left(\delta_{\nwarrow} v_{ij}^{n+1}\right)\,\left(\delta_{\nearrow} u_{ij}^{n+1}\right) + \left(\delta_{\nearrow} v_{ij}^{n+1}\right)\,\left(\delta_{\nwarrow} u_{ij}^{n+1}\right)\right]$$



Semi-implicit continuity equation ('predictor-corrector')

Inconsistent continuity equation

$$\overline{\rho}^{n+1} = \overline{\rho}_{exp}^{n+1} + \frac{\Delta t}{2}\rho^{ref}\left(\tilde{\mathbb{D}}_k^{n+1} - D^{n+1}\right), \qquad (14)$$

where D is Eulerian divergence based on the discretization of

$$D = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}.$$

Use (14) and correct error at previous time level:

$$\overline{\rho}^{n+1} = \overline{\rho}_{exp}^{n+1} + \frac{\Delta t}{2}\rho^{ref}\left(\tilde{\mathbb{D}}^{n+1} - D^{n+1}\right) - \frac{\Delta t}{2}\rho^{ref}\,\overline{\left(\mathbb{D}^{n} - D^{n}\right)\frac{\delta A_*^n}{\Delta A}}.$$

+ use traditional semi-Lagrangian grid-point scheme for u,v,T equations

Much simpler elliptic equation

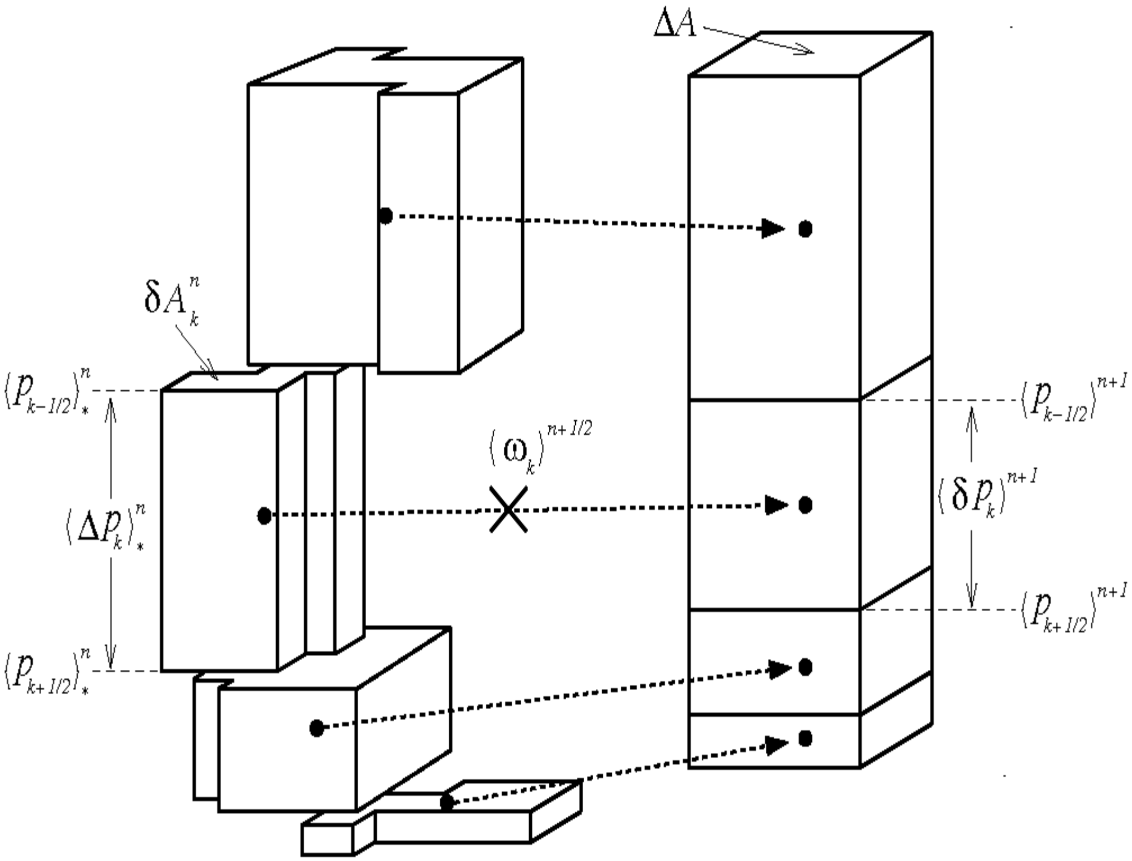
Model tested in limited area shallow water setup in Lauritzen et. al (2006, Mon. Wea. Rev.)

Part III:

Vertical problem

Floating Lagrangian coordinates

Vertical extension of Lagrangian cells



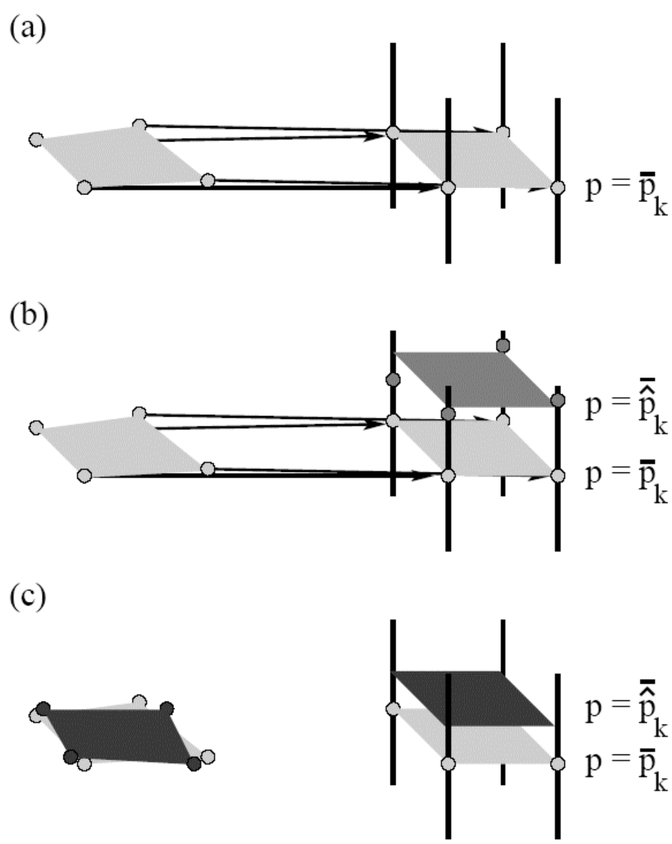
- Cells move with vertical walls

- Cells depart from model levels (horizontally backward trajectories)

-The Lagrangian cells do not necessary arrive at a model layer (=Lagrangian Vertical coordinate)

-The horizontal approximation to the departure cells drawn on the Figure is that of the Nair et al. (2002) scheme.

Non-traditional trajectory algorithm



Trajectories are backward in the horizontal and forward in the vertical.

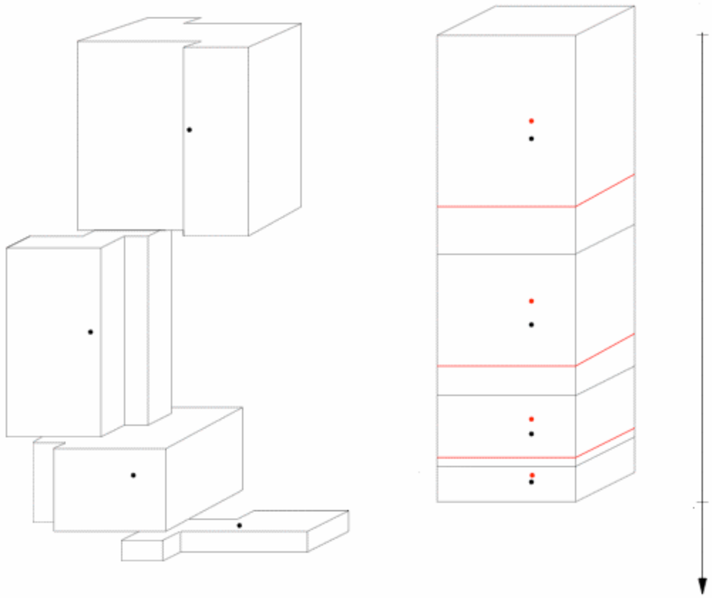
A. First guess. All computations in a model layer

B. Solve continuity equation to get pressure level thicknesses implied by CISL continuity equation

C. interpolate winds in the vertical to that level. Iterate if necessary.

VERTICAL DISPLACEMENTS ARE CONSISTENT WITH THE EXPLICIT CONTINUITY EQUATION AND HYDROSTATIC BALANCE

Implied grid and model grid



Vertical remap at every time-step
(note that long semi-implicit time-steps are used)

- Vertical grid implied by semi-implicit CISL continuity equation
- Model vertical grid: $p_{k+1/2} = A_{k+1/2} + B_{k+1/2}\, p_s^{n+1}$

$\Rightarrow$ need vertical remapping from implied grid to model grid

(use cubic Lagrange interpolation for u,v,T and PPM for “mass-related” variables)

Novel treatment of vertical velocity

For thermodynamic equation

$$\frac{dT}{dt} = \frac{R}{c_p}\frac{\omega}{p}$$

and in trajectory computation we need vertical velocity:

In traditional models ω is computed using Eulerian formula

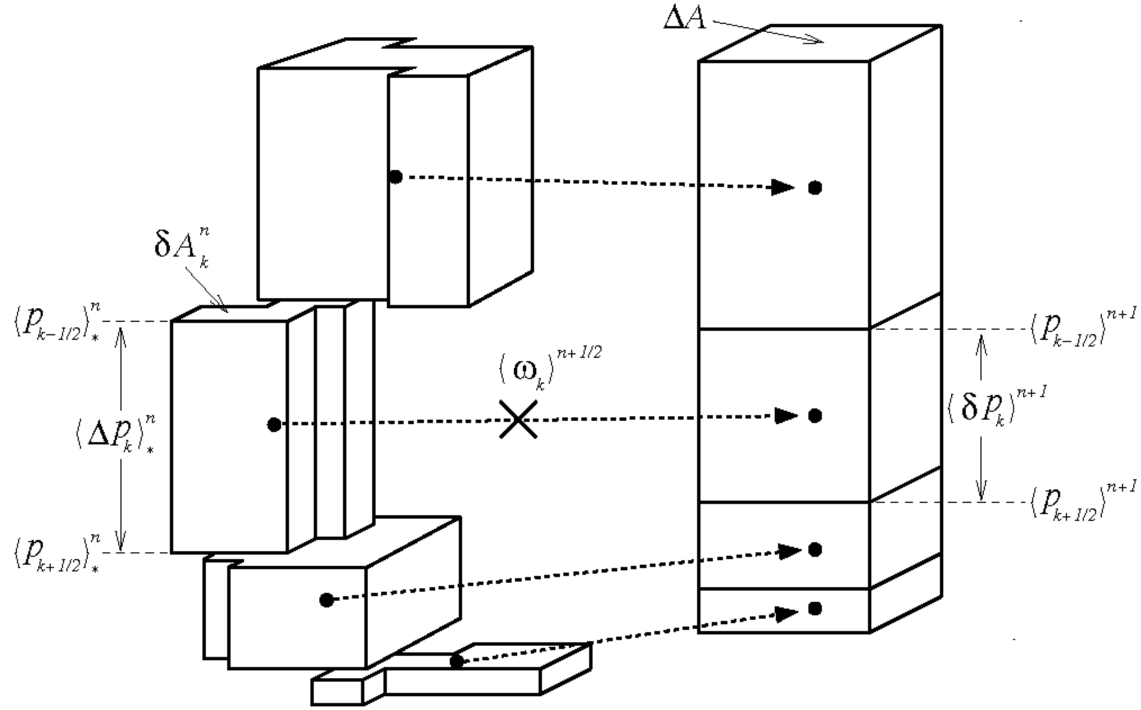
$$\omega = -\int_0^\eta \nabla \cdot \left(\vec{v}\,\frac{\partial p}{\partial \eta} \right) d\eta + \vec{v} \cdot \nabla p.$$

Vertical velocity discretized in Eulerian fashion and is not consistent with the discretized continuity equation.

It would be more consistent to use Lagrangian form

$$\omega = \frac{dp}{dt}$$

Consistent ω averaged over "mid-cell"



- **Energy conversion term and vertical displacement of cells is consistent with horizontal flow (i.e. discretized CISL continuity equation).**

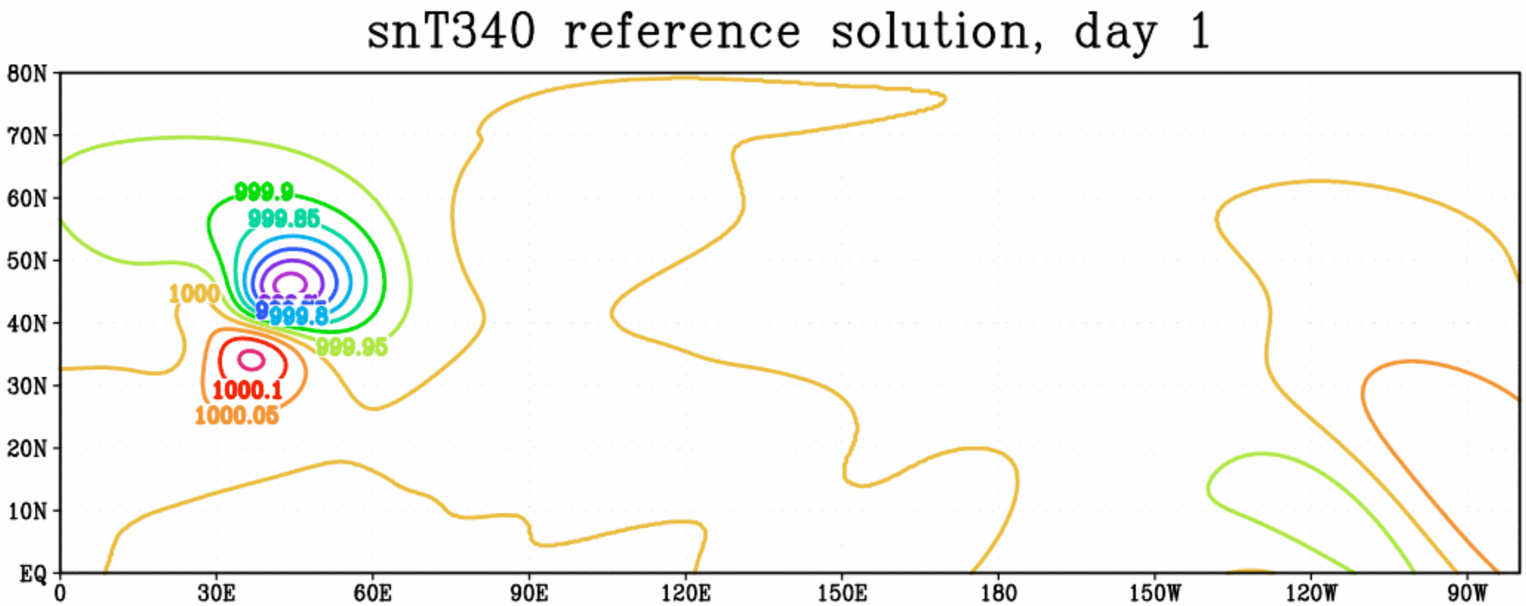
Part IV:

Preliminary testing

Adiabatic and full physics

Jablonowski-Williamson test case (for global dynamical cores)

- Initial condition is a balanced, steady-state solution before an overlaid perturbation is introduced; $p_s(t=0) = 1000$ hPa.
- Perturbation triggers the growth of a baroclinic disturbance over the course of several days.



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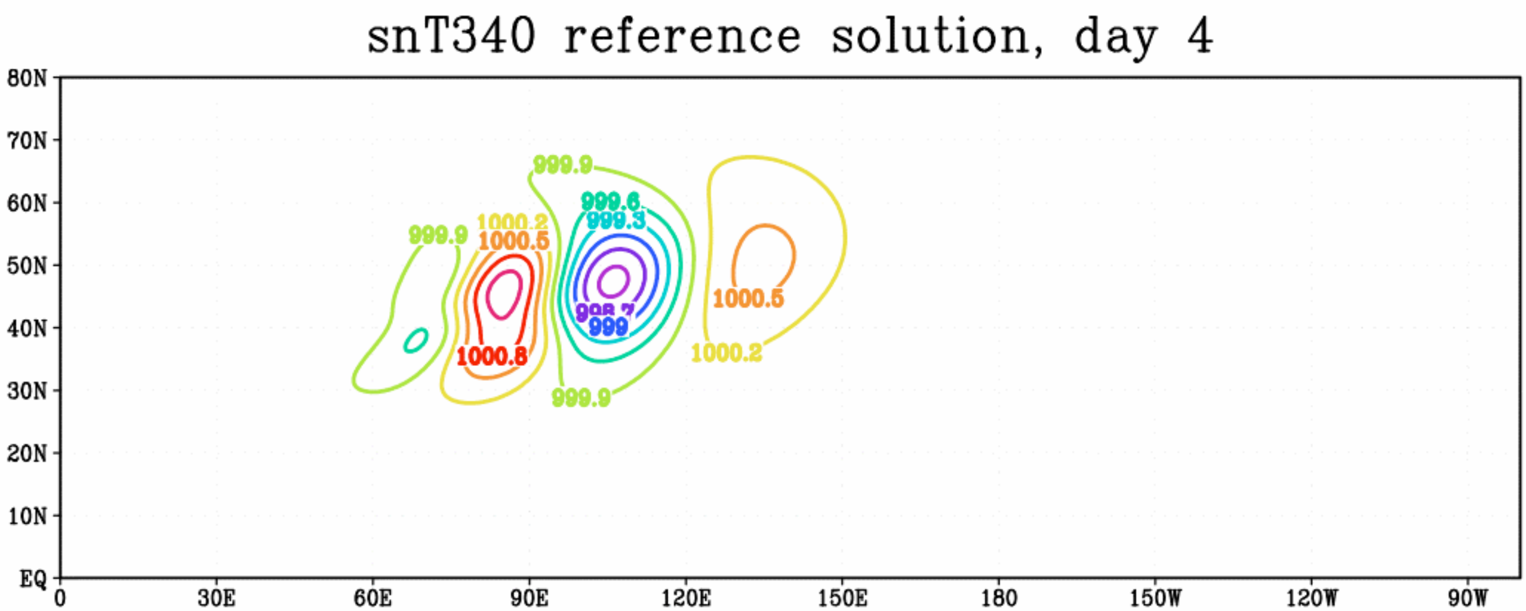

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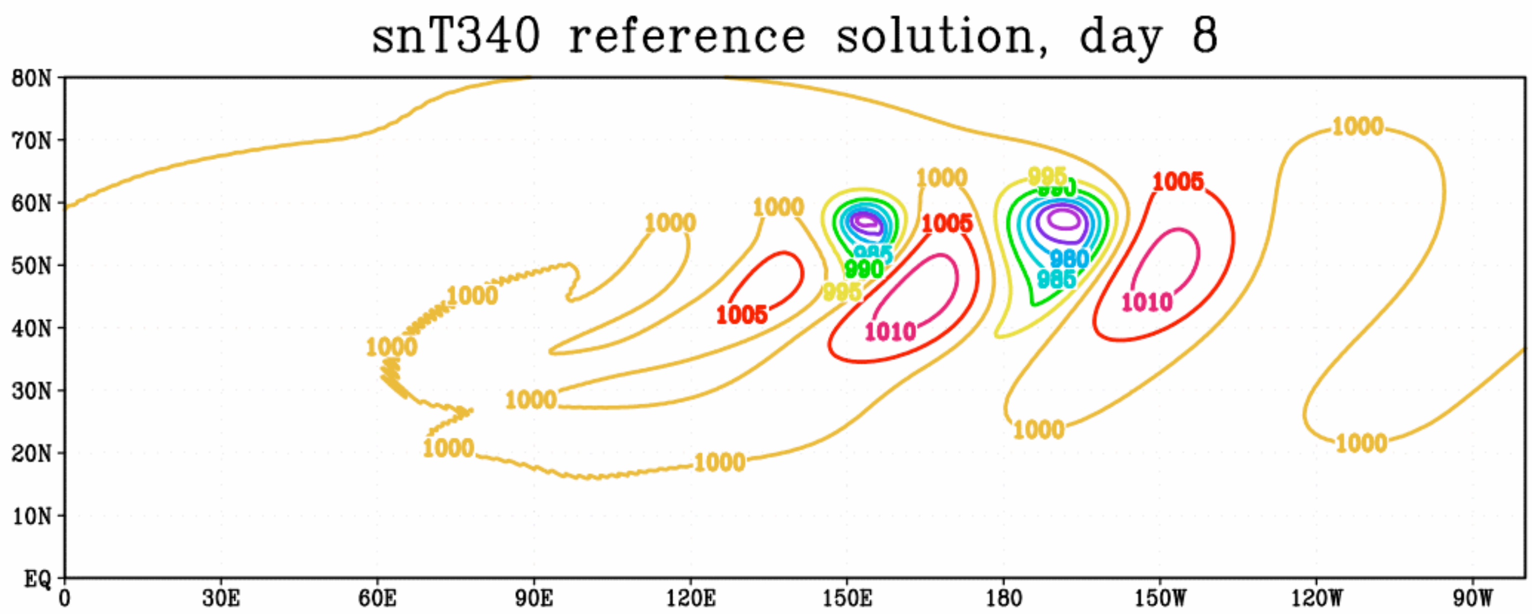
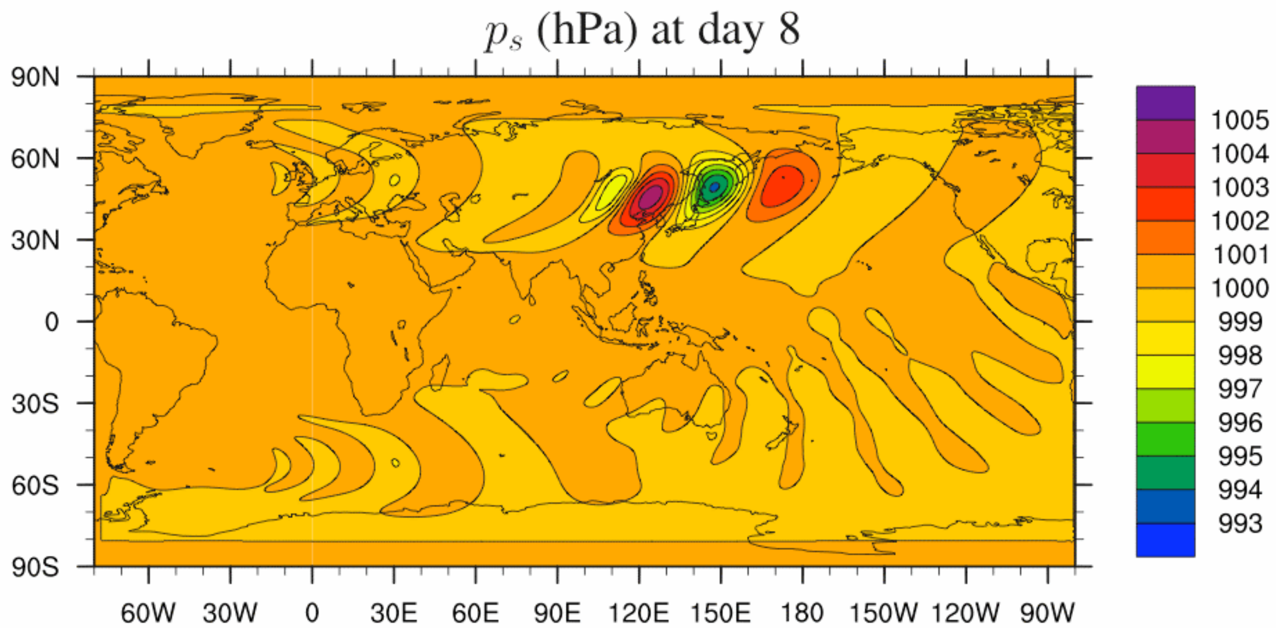


Global Jablonowski-Williamson test case adapted to limited area model domain

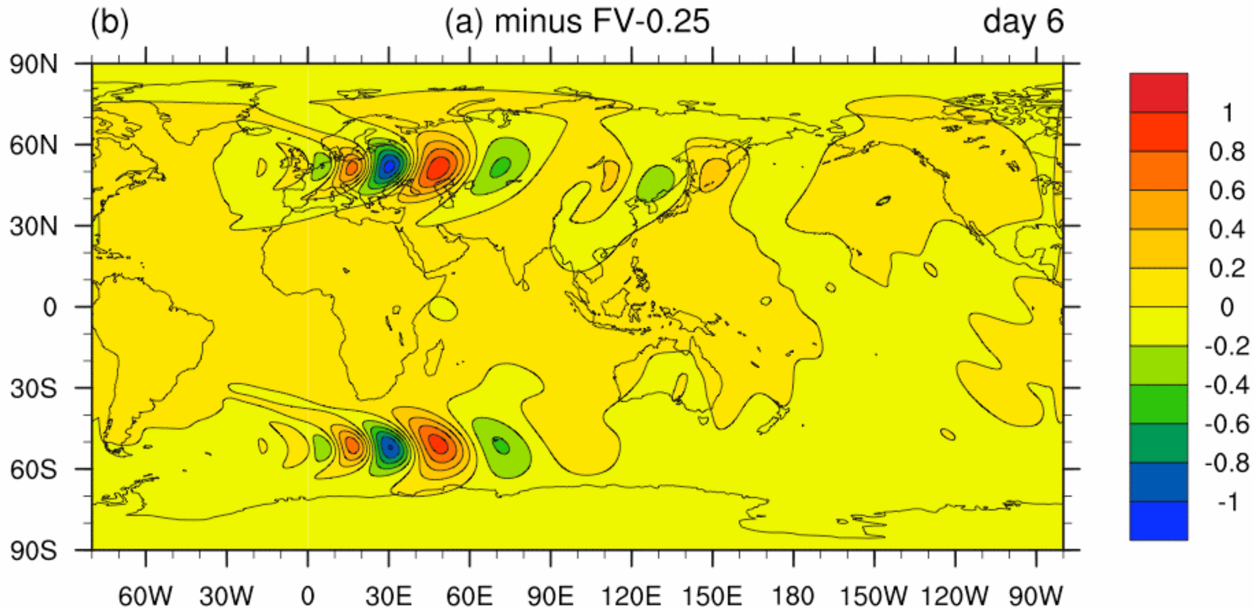
- Active domain: zonally 360° (non-periodic) and $\theta = \pm 75^\circ$
- Solution held fixed at initial condition on lateral boundaries (elliptic solver assumes zero divergence on the boundary)
- "stretching" limited-area model approximations to the limit



Global Jablonowski-Williamson test case adapted to limited area model domain

- Boundary relaxation and elliptic solver trigger low amplitude spurious wave behind main wave train!
- Exactly same spurious wave structure is seen in unperturbed run
- Possible to 'filter it out'

p_s (hPa) at day 8



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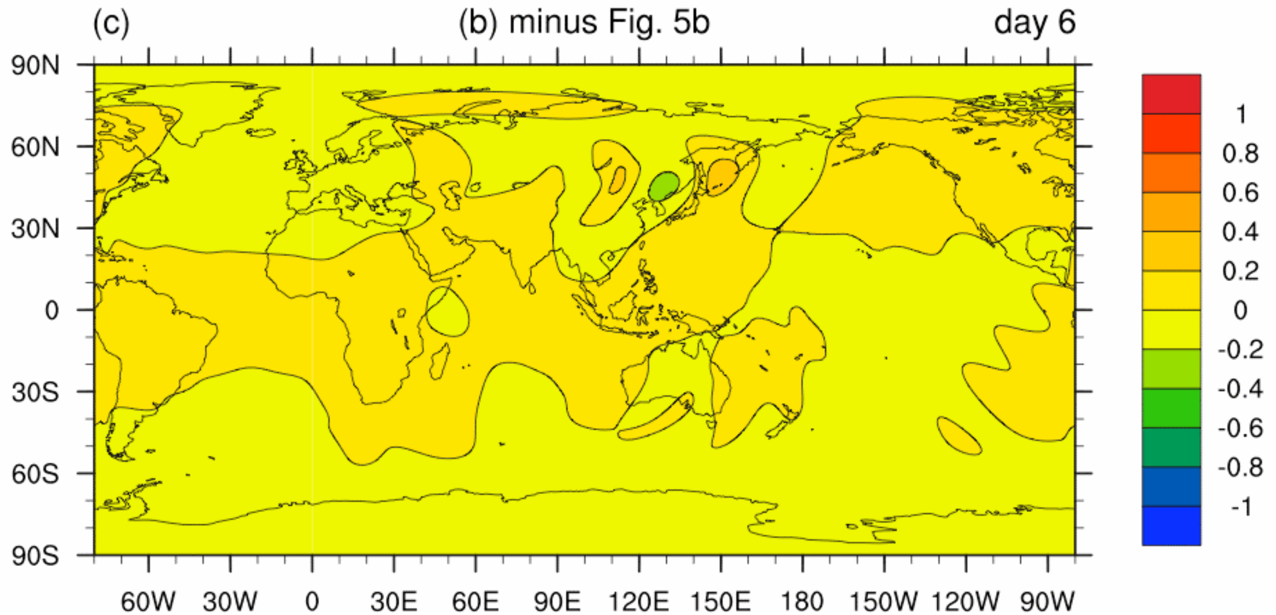
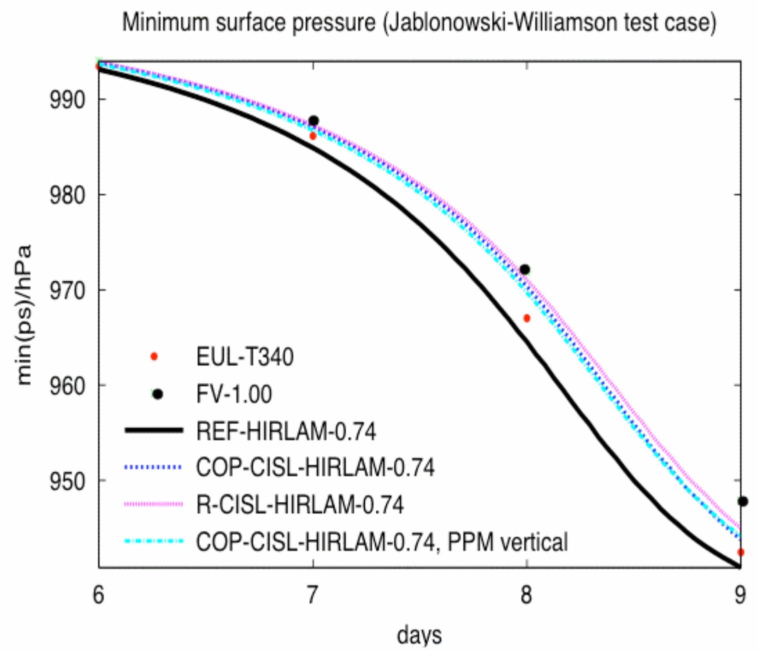
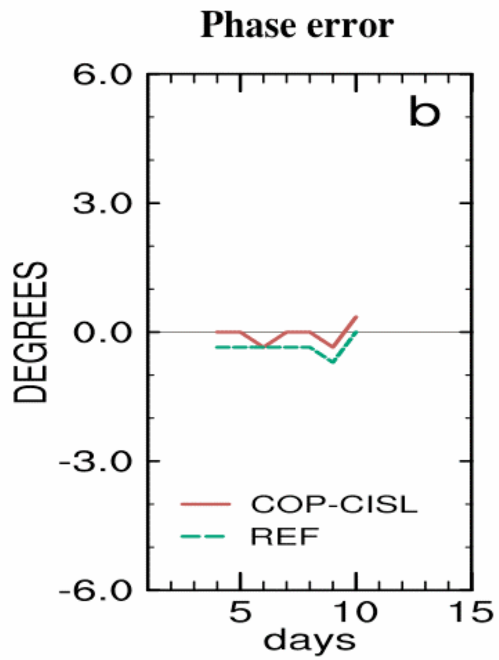


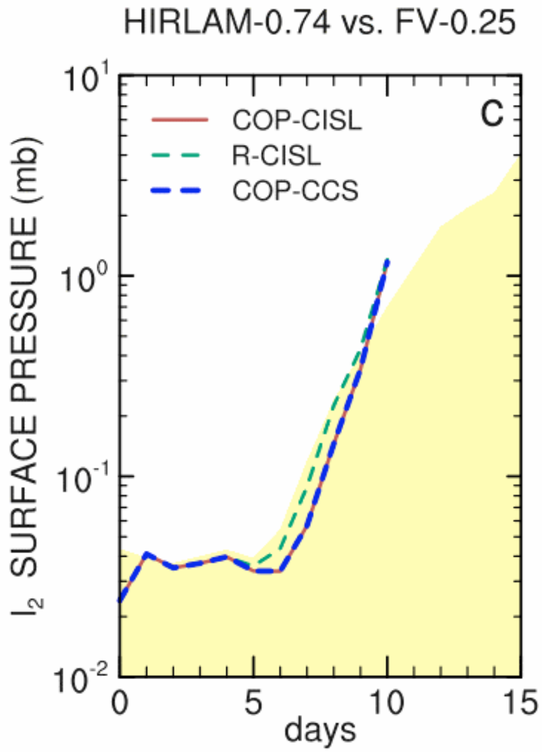
Effect of vertical remapping method





- REF-HIRLAM tendency to over-developing baroclinic wave: Phase errors in REF-HIRLAM?
- Vertical remapping is source of internal diffusion in CISL-HIRLAM.

l_2 differences: Effect of inconsistent conversion term



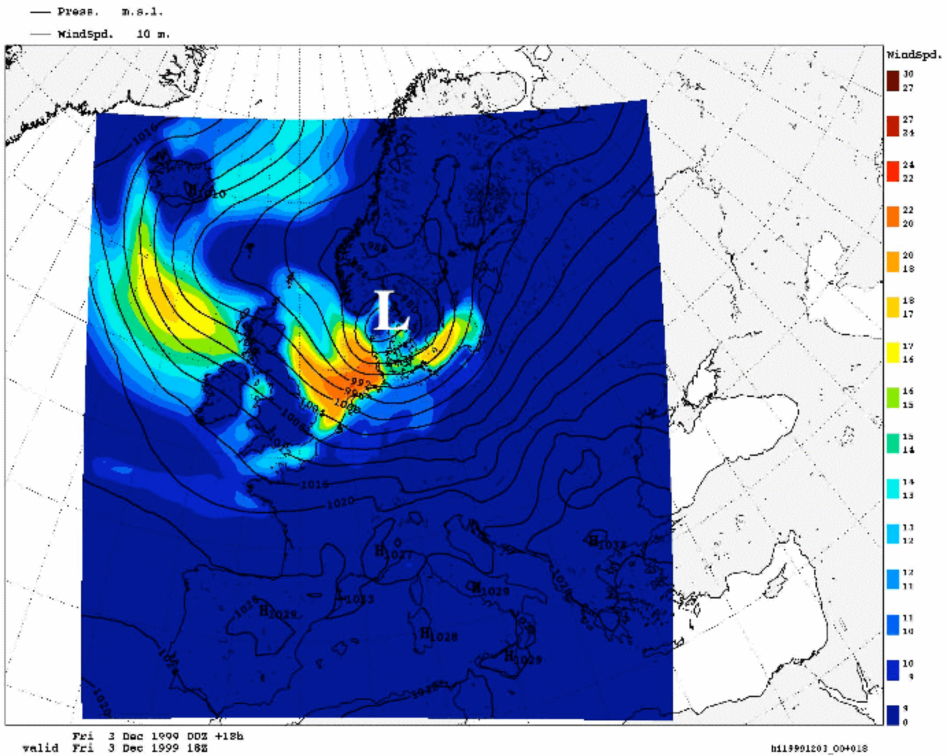
- R-CISL-HIRLAM : Inconsistent (traditional) conversion term discretization
- COP-CISL: CISL-HIRLAM based on fully 2D CISL advection scheme
- COP-CCS: CISL-HIRLAM based on cascade advection scheme (2D problem split into 1D problems)

Two horizontal resolutions are used. The lower resolution is $\Delta\lambda \approx 1.45°$, $\Delta\theta \approx 1.15°$, and the highest resolution is $\Delta\lambda \approx 0.74°$, $\Delta\theta \approx 0.59°$. In the vertical there are 27 levels and the placement of the levels is as in JW06, but with one more layer added at the top of the atmosphere (so that the pressure at the upper boundary is zero as in HIRLAM). The time-step for the low- and high-resolution runs is 30 and 15 minutes, respectively.

- As expected R-CISL-HIRLAM less accurate (this also holds for phase errors)
- Suggests that consistent discretization of energy conversion term in the thermodynamic equation is important for strong baroclinic developments.
- COP-CISL and COP-CCS are indistinguishable

COP-CISL-HIRLAM with physics: Storm case

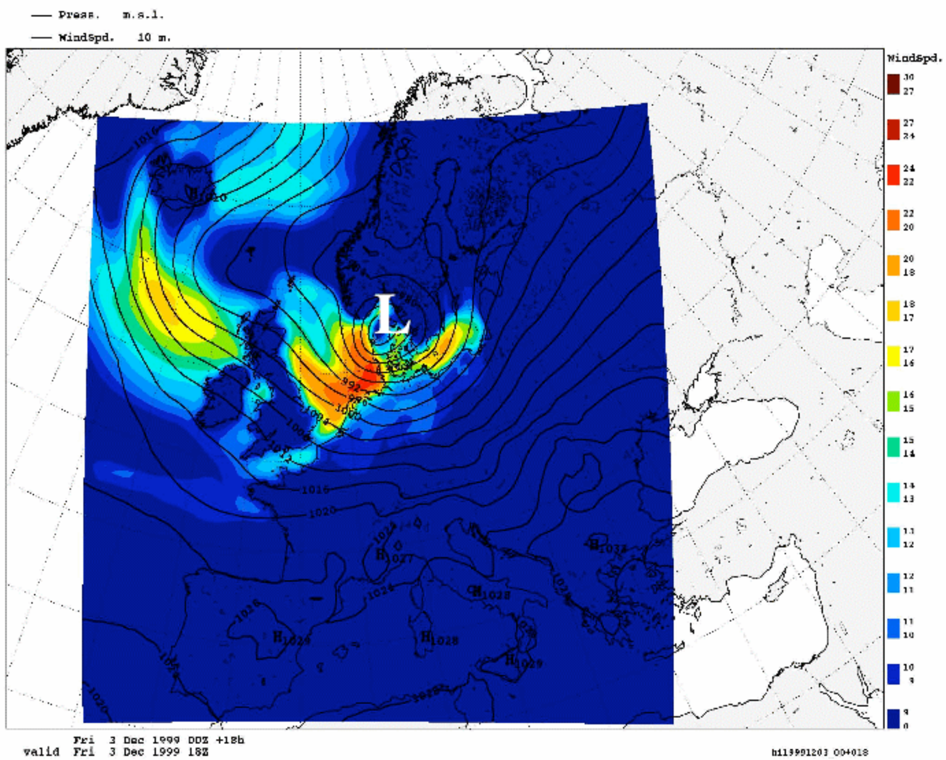
Coarse resolution: $(\Delta\lambda, \Delta\theta) = (0.4°, 0.4°)$, min($p_{mslp}$)=972 hPa



Standalone 18h forecast (no data assimilation); No tuning of physics

Reference HIRLAM with physics: Storm case

Coarse resolution: $(\Delta\lambda, \Delta\theta) = (0.4°, 0.4°)$, min($p_{mslp}$)=968 hPa



Ongoing research project with Danish Meteorological Institute

Summary

- Model stable for long time-steps.
- Convergence in Jablonowski-Williamson baroclinic wave test case.
- For relatively low resolution runs finite-volume dynamical cores seem to need a higher resolution for obtaining same accuracy compared to spectral dynamical cores (not shown).
- Indication that consistent discretization of energy conversion term in thermodynamic equation is important during strong baroclinic developments.
- Efficiency: In current implementation CISL models approx. twice as expensive (however, ad hoc coding compared with optimized HIRLAM).
- Model can perform online transport with a very high level of consistency!

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See http://www.cgd.ucar.edu/cms/pel/publications.html

Questions